ROLL NO.____________________ **Class: XII (2019-20)**

Time : 3hrs **MATHEMATICS (041)** M.M. : 80

(i) All questions are compulsory
(ii) The question paper consists of 36 questions divided into four sections A, B, C and D. Section A comprises of 20 questions of one mark each. Section B comprises of 6 questions of two marks each. Section C comprises of 6 questions of four marks each and Section D comprises of 4 questions of six marks each.
(iii) Use of calculators is not permitted.

SECTION –A

1. If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, find α satisfying $0 < \alpha < \frac{\pi}{2}$ when $A + A^T = \sqrt{2} I_2$.

2. Let $A = \{1, 2, 3, 4\}$. Let R be an equivalence relation on AXA defined by $(a, b)R(c, d)$ iff $a + d = b + c$d=b+cd equivalence class $[(1, 3)]$.

3. Evaluate: $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

4. Using derivative, find the approximate percentage increase in the area of a circle if its radius is increased by 2%.

5. Determine the value of constant 'k' such that the function $f(x) = \begin{cases} \frac{kx}{|x|} & if \quad x < 0 \\ 3 & if \quad x \geq 0 \end{cases}$ is continuous at $x = 0$.

6. If $\begin{bmatrix} x + y \\ x - y \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. Find (x, y)

7. Write the derivative of $|x - 5|$ at $x = 2$.

8. Evaluate: $\sin^{-1}\left(2\cos\left(-\frac{3}{5}\right)\right)$.

9. Find the point on the curve $y^2 = 8x + 8$ for which the abscissa and ordinate change at the same rate.

10. If A and B are square matrices of order 3 each, $|A| = 2$ & $|B| = 3$. Find $|3AB|$.

11. Solve for x: $\tan^{-1} x = \sin^{-1}\frac{1}{\sqrt{2}}$.

12. If $f(x) = x + 7$ and $(x) = x - 7$; $x \in \mathbb{R}$, then find $\frac{d}{dx}(fog)(x)$.

13. Given $A = [a_{ij}]_{3X3}$ where $a_{ij} = i + 3j$. Write the elements (i) a_{12} (ii) a_{24} .

14. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$.

15. Give an example of a skew symmetric matrix of order 3.

16. Find the derivative of $f(e^{\tan x})$ w.r.t. x at $x = 0$. It is given that $f'(1) = 5$.

17. If A is a square matrix with $|A| = 4$. Find the value of $|A.(adj\ A)|$.

18. Write the smallest reflexive relation on the set $A = \{2, 3, 5\}$.

19. Using Matrices, Examine the system of equations for consistency or inconsistency:

$x + 3y = 5;\ 2x + 6y = 8.$

20. Find the differential of the function $\cos^{-1}(\sin 2x)$.

SECTION –B

21. A kite is moving horizontally at a height of 151.5 m. If the speed of kite is 10 m/s, how fast is the string being let out when the kite is 250 m away from the boy who is flying the kite? The height of boy is 1.5 m.

22. Using Determinants, prove that $a + b = ab$, if the points $(a, 0)$ and $(0, b)$ lie on the line passing through the point $(1, 1)$.

23. Discuss the continuity of the function $f(x) = \begin{cases} [x-2] + [2-x] & , \quad x \neq 2 \\ 0 & , \quad x = 2 \end{cases}$ at $x = 2$.

24. Find the matrix X such that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

25. Prove that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$.

26. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time an 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. Formulate the LPP

SECTION –C

27. Show that the relation R defined on set A of all polygons as $R = \{(P_1, P_2) \mid P_1\ \&\ P_2$ have same number of sides$\}$ is equivalence relation. What is the set of all elements in A related to right angled triangle T with sides 3, 4 and 5?

28. Verify Rolle's Theorem for the function $f(x) = x^3 + 3x^2 - 24x - 80$ on $[-4, 5]$.

29. Solve for x: $\tan^{-1}\left(\frac{x-2}{x-3}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \frac{\pi}{4}$; $|x| < 1$.

30. Express $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices.

31. Using Properties of Determinants, Prove that:

$$\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$$

32. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ if $x \in (-1, 1); x \neq 0$.

SECTION –D

33. Consider $f: \boldsymbol{R} \to [-5, \infty)$ defined by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$. Hence, find (i) $f^{-1}(10)$ (ii) y if $f^{-1}(y) = \frac{4}{3}$.

34. Two schools A and B want to award their selected students on the values of Sincerity, Truthfulness and Helpfulness. The School A wants to award ₹ x each, ₹y each and ₹ z each for the three respective values 3, 2 and 1 students with a total award money of ₹ 1600. School B wants to spend ₹ 2300 to award its 4, 1 and students on the respective values (by giving the same award money for the three values as before). If the total amount of awards for one prize on each value is ₹ 900, using Matrices, find the award money for each value.

35. Find the value of p when the curves $x = 9p(9 - y)$ and $x^2 = p(y + 1)$ cut each other at right angles.

36. A farmer mixes two brands p and Q of cattle feed. Brand P, costing ₹ 250 per bag, contains 3 units of nutritional elements A, 2.5 units of element B and 2 units of element C. Brand Q costing ₹ 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

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